ECE4390 Lab 4

The purpose of this lab is to familiarize the student with the Finite Element Method (FEM). This computational technique will be applied to solve the 2D Laplace ($\nabla^2 \Phi(x, y) = 0$) and Poisson ($\nabla^2 \Phi(x, y) = -\rho/\epsilon$) equations. GMSH will be used to create the meshes for the problem domains. GMSH is distributed under the terms of the GNU General Public License and is available for free at http://geuz.org/gmsh/. A good introductory GMSH tutorial is available at http://ffep.sourceforge.net/Download/gui_tutorial.pdf.

A report is to be handed-in. All the requested plots are to be submitted. Be sure to include any explanations describing the plots as needed. Also, submit a print-out of your code but if you wish to save paper, the files can be emailed to umfauch2@cc.umanitoba.ca. Be sure to include your name and student number. There will also be a 15 to 20 minute quiz to be written near the end of the lab period.

Part 1: Familiarization with the FEM code

1a. Download the laboratory files from the course website into a single folder. Run the "electrostatic_fem_probe.m" file in MATLAB. Observe the result. Go through the source code and locate the lines that set the Dirichlet boundary condition values. Spend some time to understand how the global system of equations is created to solve Laplace's equation from the given mesh.

1b. Run the program for the same domain but with the upper boundary set to a Dirichlet condition of $\Phi = 50$. Do this by changing the physics number of the upper boundary in the probe geometry file. Use GMSH to create a mesh with at least 6500 triangular elements over the new geometry. Use the "electrostatic_fem_probe.m" program to find the solution to the new domain. Be sure to add appropriate code to handle the new type of boundary. Plot the result of the above modifications.

Part 2: Solution of Laplace's equation using FEM

2a. Using GMSH, create a mesh for the domain shown below:
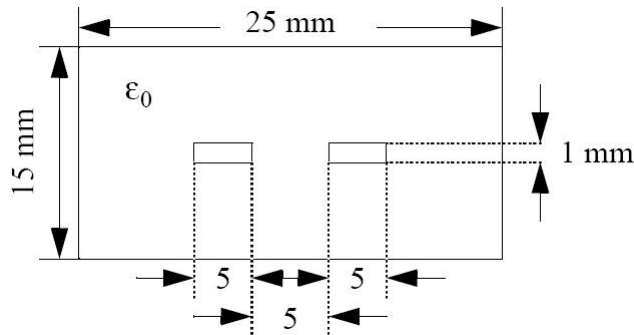
Set the outer boundary physics number to 100, the left inner-box boundary to 200 and the other to 201. Refine the mesh such that it is composed of at least 6500 elements.

2b. Solve Laplace's equation over the domain described above with the outer boundary set to $\Phi = 0$. Also, as was done in the previous lab, set boundary conditions on the inner boxes to represent two conductors. Set the left conductor to 1V and the other to ground. Plot the result.

2c. Use the FEM solution to determine the p.u.l. capacitance of the transmission line. This is done by applying the integral form of Gauss's law for a contour of your choice about the left conductor. Use the contour integration method from the previous lab as a guidleline for this case. Give the details about how you decided to implement this integral. Your result should closely match the p.u.l. capacitance found in the previous lab. Hint: Recall from class notes that $\Phi(x, y)$ for (x, y) in the problem domain is approximated by $\phi^{(e)}(x, y) = \underline{\phi}^T \underline{\alpha}(x, y)$ where e is the element that contains the point (x, y), $\underline{\phi}$ is a column vector of solution values at the vertices of element e and $\underline{\alpha}$ is the column vector of shape functions for element e. To find the element that contains a particular point, the provided MATLAB function "getEleIndices.m" can be used. It requires two input parameters. The first is the struct that is returned by the call to "GmshReadM.m". The second parameter is a $p \times 2$ matrix where p is the number of points to locate and each point is described by its x-coordinate in the first column and its y-coordinate in the second column. The function then returns a $p \times 1$ vector of element indices in order of corresponding points.

Part 3: Solution of Poisson's equation using FEM

The geometry shown below represents a charged plate enclosed by a grounded boundary. A Gaussian function centered at the origin describes the charge distribution on the plate. No free charge is imposed outside the plate. Succinctly, imposed charge within the entire domain is given by

$$\rho = g(x,y) = \begin{cases} 100e^{-(100x^2 + 100y^2)} & \text{if } (x,y) \in \Sigma \\ 0 & \text{otherwise} \end{cases}$$

3a. Using GMSH, create a mesh for the domain shown below:
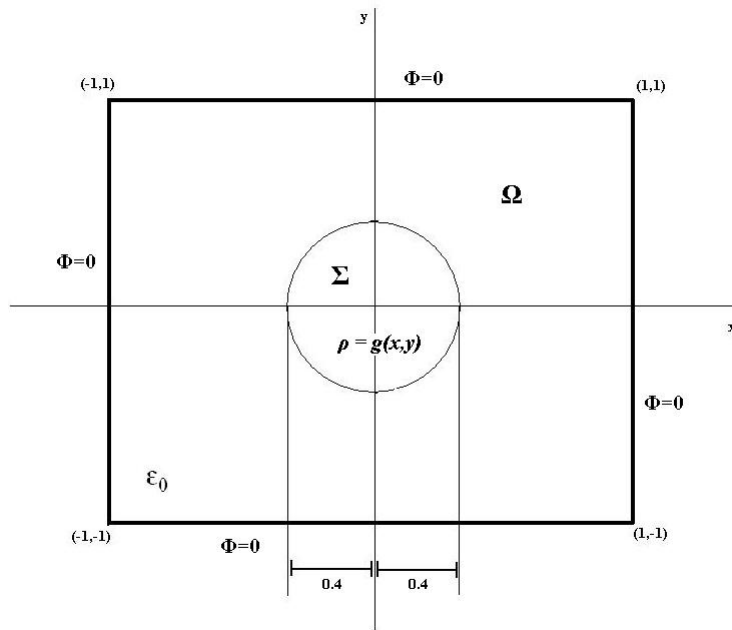
Set the outer boundary physics number to 100, the Ω area physics number to 200 and the Σ area to 201. Refine the mesh such that it is composed of at least 6500 elements.

3b. Modify the given FEM program to solve Poisson's equation such that the forcing function $\rho = g(x,y)$ is evaluated as a constant value over a particular element. For each element, $g(x,y)$ should be evaluated at the element centroid. Plot the solution. Determine the total charge on the plate by applying the integral form of Gauss's law. Hint: The element centroid coordinates are available in the form of two vectors in the "MeshData" struct that is returned from a call to "GmshReadM.m". The vectors of x and y coordinates are named "xCentroids" and "yCentroids", respectively. In these vectors, the coordinates are ordered by node index. The 2D element physics numbers are also available in the "MeshData" struct. They are ordered by node index in the "PhysNumber" vector. It is also useful to note that the integral of a shape function over its element is equal to a third of the element area.

3c. Modify the given FEM program to solve Poisson's equation such that the forcing function $\rho = g(x,y)$ is expanded with shape functions over each element.

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Plot the solution. Determine the total charge on the plate by applying the integral form of Gauss's law.

3d. Compare the two computed solutions. Which is more accurate and why?